1. Plan

Objectives
1. To graph absolute value functions

Examples
1. Graphing an Absolute Value Function
2. Using a Graphing Calculator
3. Writing Two Linear Equations
4. Real-World Connection

Math Background
An absolute value function is a special kind of piecewise function. The pieces can be determined by applying the definition of absolute value.

More Math Background: p. 52D

Lesson Planning and Resources
See p. 52E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You’ll Need
Lesson 2-2 and Skills Handbook page 879

Graph each equation for the given domain and range. 1–2. See below left.
1. \(y = x\) for real numbers \(x\) and \(y\) \(\geq 0\)
2. \(y = 2x - 4\) for real numbers \(x\) and \(y\) \(\geq 0\)
3. \(y = -x + 6\) for real numbers \(x\) and \(y\) \(\leq 3\) See back of book.

New Vocabulary
• absolute value function
• vertex

Graphing Absolute Value Functions

A function of the form \(f(x) = |mx + b| + c\), where \(m \neq 0\), is an absolute value function. The related equation \(y = |mx + b| + c\) is an absolute value equation in two variables. Graphs of absolute value equations in two variables look like angles.

The vertex of a function is a point where the function reaches a maximum or minimum. In general, the vertex of \(y = |mx + b| + c\) is located at \((-\frac{b}{m}, c)\). In the middle graph above, the \(x\)-coordinate of the vertex is \(-\frac{2}{3}\) = 2.

EXAMPLE

Graphing an Absolute Value Function

Graph \(y = |3x + 12|\).

Evaluate the equation for several values of \(x\), beginning with \(x = -\frac{b}{m} = -\frac{12}{3} = -4\). Make a table of values.

| \(x\) | -6 | -4 | -3 | -2 |
| \(y\) | 6  | 0  | 3  | 6  |

Graph the function.

Differentiated Instruction

Special Needs
Write "absolute Value functions" on the board and ask students to describe the shape of their graphs. Emphasize that the graph can open upward or downward, with the vertex as either a minimum or maximum, respectively.

Below Level
Help students understand that, to create a table for the graph, they must choose \(x\)-values both less than and greater than the vertex \(\frac{b}{m}\).
Graph each equation.  a–b. See margin.

a. \( y = |2x - 5| \)  \hspace{1cm} b. \( y = -|x + 1| - 2 \)

You can use a graphing calculator to graph an absolute value equation.

**EXAMPLE**  Using a Graphing Calculator

Graph \( y = -|3x + 4| + 6 \) on a graphing calculator.

Use the absolute value function.

Graph the equation \( Y1 = -\text{abs}(3X + 4) + 6 \).

**EXAMPLE**  Writing Two Linear Equations

Graph \( y = |x - 3| + 5 \).

**Step 1** Isolate the absolute value.

\[
\begin{align*}
y &= |x - 3| + 5 \\
y - 5 &= |x - 3|
\end{align*}
\]

**Step 2** Use the definition of absolute value. Write one equation for \( x - 3 \geq 0 \) and a second equation for \( x - 3 < 0 \).

\[
\begin{align*}
\text{when } x - 3 &\geq 0 \quad \text{when } x - 3 < 0 \\
y - 5 &= x - 3 \\
y - 5 &= -(x - 3) \\
y &= x + 2 \\
y &= -x + 8
\end{align*}
\]

**Step 3** Graph each equation for the appropriate domain.

When \( x - 3 \geq 0 \), or \( x \geq 3 \),
\( y = x + 2 \).

When \( x - 3 < 0 \), or \( x < 3 \),
\( y = -x + 8 \).

Graph each equation by writing two equations.

a. \( y = \frac{3}{2}x + 4 - 3 \)  \hspace{1cm} b. \( y = 2 - |x + 1| \)

a–b. See back of book.
You can use absolute value functions to model time and distance problems. You can consider the time before you arrive at a destination to be negative.

**EXAMPLE Real-World Connection**

**Travel** Suppose you pass the Betsy Ross House halfway along your trip to school each morning. You walk at a rate of one city block per minute. Sketch a graph of your trip to school based on your distance and time from the Betsy Ross House.

The graph would stretch upward.

**a. Critical Thinking** Suppose you ride your bicycle to school at a rate of three city blocks per minute. How would the graph of your trip to school change? 

**b. Sketch a new graph.**

**EXERCISES**

For more exercises, see Extra Skill and Word Problem Practice.

**Practice and Problem Solving**

**Practice by Example**

1. $y = |4x|$
2. $y = |4x| - 1$
3. $y = |4x - 1|$
4. $y = |-3x|$
5. $y = |-3x| + 2$
6. $y = |-3x + 2|$
7. $y = -|2x|$
8. $y = -|2x| + 5$
9. $y = -|2x + 5|$

1–18. See back of book.

Graph each equation on a graphing calculator. Then sketch the graph.

10. $y = |x + 2| - 4$
11. $y = 4 - |x + 2|$
12. $y = 4|x + 2|$
13. $y = \frac{1}{3}|3 - 3x|$
14. $y = \frac{1}{3} - \frac{1}{3}|x|$
15. $y = \frac{3}{2}|x| - \frac{5}{2}$
16. $y = |x| + \frac{1}{2}|x|$
17. $y = \frac{1}{2}|x| - |x|$
18. $y = \frac{1}{2}|x - \frac{1}{2}|$

**Example 3** (page 89)

**Graph each equation by writing two linear equations.** 19–28. See back of book.

19. $y = |x + 6|$
20. $y = |3x + 6|$
21. $y = |3x - 6|$
22. $y = -|x - 5|$
23. $y = |2x + 1|$
24. $y = \frac{3}{2}|3x - 1|$
25. $y = |x - 2| - 6$
26. $y = \frac{1}{2}x - 4 + 4$
27. $y = \frac{1}{2}|x + 2| - 2$
28. **Manufacturing** The conveyor belt at a factory operates continuously 24 hours a day, carrying vitamin bottles and moving two feet each minute. Sketch a graph showing the distance in feet from the filling arm of one bottle on the conveyor belt before and after it is filled. Use the x-axis for time before and after the bottle is filled and the y-axis for distance from the filling arm.
Lesson 2-5 Absolute Value Functions and Graphs

Graph each absolute value equation. 33–50. See back of book.
33. \( y = |4x + 2| \) 34. \( y = | -3x + 5| \) 35. \( y = |4 - 2x| \)
36. \( y = -\frac{1}{2}x - 1 \) 37. \( y = \frac{1}{2}x - 2 \) 38. \( y = \frac{3}{2}x + 2 \)
39. \( y = |3x - 6| + 1 \) 40. \( y = | -x - 3| \) 41. \( y = |2x + 6| \)
42. \( y = |2x + 2| - 3 \) 43. \( y = 6 - |3x| \) 44. \( y = 6 - |3x + 1| \)
45. \( y = | -2x - 1 + 1 \) 46. \( y = 2|x - 3| \) 47. \( y = -\frac{3}{2} \frac{1}{2}x \)
48. \( 2y = \frac{1}{2}|x + 2| \) 49. \( \frac{1}{2}y - 3 = -|x + 2| \) 50. \( -3y = |3x - 6| \)

51. Multiple Choice The graph at the right models a car traveling at a constant speed. Which equation best represents the relation shown in the graph?
A. \( y = |60x| \)
B. \( y = |x + 60| \)
C. \( y = |60 - x| \)
D. \( y = |x| + 60 \)

52. a. Graph the equations \( y = \frac{1}{2}x - 6 \) + 3 and \( y = -\frac{1}{2}x + 6 \) - 3 on the same set of axes. a–b. See back of book.

b. Writing Describe the similarities and differences in the graphs.

53. \( y = |3x - x| \) 54. \( y = x - 2|x| \) 55. \( y = 2x | - x \)
56. \( y = \frac{1}{2}|x - 3| + 5 \) 57. \( y = \frac{1}{2}|x + 4| \) | x - 1 | 58. \( y = |x + 1| + |x| \)

59. a. Open-Ended Find two absolute value equations with graphs that share a vertex. \( y = |x|, y = -|x| \)
b. Find two absolute value equations with graphs that share part of a ray. \( y = |x|, y = |x - 1| + 1 \)
The graph at the right models which equation?

A. \( y = |3x - 1| + 2 \)  
B. \( y = |x - 1| - 2 \)  
C. \( y = |x - 1| + 2 \)  
D. \( y = |3x - 3| - 2 \)

61. What is the vertex of \( y = |x| - 5 \)?
   A. (5, 0)  
   B. (0, -5)  
   C. (0, 5)  
   D. (0, 10)

62. What is the vertex of \( y = -|x| - 2 \)?
   A. (0, 0)  
   B. (0, 1)  
   C. (0, 2)  
   D. (0, 2)

63. What is the vertex of \( y = |x - 3| + 5 \)?
   A. (0, 5)  
   B. (0, 3)  
   C. (3, 5)  
   D. (3, 0)

64. Which pair of linear equations represents the equation \( y = |x + 3| - 4 \)?
   A. \( y = x + 1 \) for \( x \geq 3 \)  
   B. \( y = x - 1 \) for \( x \geq 3 \)  
   C. \( y = -x - 1 \) for \( x < 3 \)  
   D. \( y = -x - 1 \) for \( x < 3 \)

65. Explain how to find the x-coordinate of the vertex of \( y = |3x - 6| \).

66. How can you graph the equation \( y = -|5x + 1| \) by writing two linear equations? Show both equations, and label the coordinates of the vertex in your graph.

Mixed Review

Lesson 2-4
Graph each set of data. Decide whether a linear model is reasonable. If so, draw a trend line and write its equation. 67–70. See back of book.

67. \([(0, -5), (5, 25), (7, 44), (9, 70), (11, 90)]\)
68. \([(0, 0), (-4, 4), (-1, 6), (2, 8), (5, 10)]\)
69. \([(0, 7), (-1, 4), (0, 5), (3, 8), (4, 7)]\)
70. \([(5, 7), (2, 6), (5, 4.5), (6, 4), (9, 2.5)]\)

Lesson 2-4
Find the slope of each line.

71. \( 3x + y = -3 \) \hspace{1cm} 72. \( 5y - 20x = 6 \) \hspace{1cm} 73. \( y = \frac{3}{2}x - \frac{1}{3} \)
74. \( 12x - 3y = 4 \) \hspace{1cm} 75. \( 2x + 3y = 1 \) \hspace{1cm} 76. \( 0.1y = 0.5x + 0.1 \) \hspace{1cm} 77. \( y = -18h; \text{linear} \)

Lesson 1-3
Solve each equation.

78. \( 17x = 187 \) \hspace{1cm} 79. \( 13x - 26 = 91 \) \hspace{1cm} 80. \( 2(a - 6) + 11 = 25 \) \hspace{1cm} 81. \( 7(b + 3) - 18(1 - b) = 103 \) \hspace{1cm} 82. \( 6(m + 3) = 3(5 - m) + 66 \)

65. [2] The vertex of \( y = |3x - 6| \) would be where \( 3x - 6 = -(3x - 6) \) because that would be \( x = 2 \). 

[1] only includes solution \( x = 2 \).