Two-Variable Inequalities

What You’ll Learn

• To graph linear inequalities
• To graph absolute value inequalities

. . . And Why

To solve problems involving combinations, as in Example 2

New Vocabulary

• linear inequality

1. Graphing Linear Inequalities

Activity: Linear Inequalities

1. Graph the line $y = 2x + 3$ on graph paper. 1–2. See back of book.

2. a. Plot each point listed below.
   $(-2, -3), (-2, -1), (-1, -1), (1, 5)$,
   $(0, 4), (0, 5), (1, 6), (2, 3), (2, 7)$
   b. Classify each point as on the line, above the line, or below the line.

3. Are all the points that satisfy the inequality $y > 2x + 3$ above, below, or on the line? above the line

A linear inequality is an inequality in two variables whose graph is a region of the coordinate plane that is bounded by a line. To graph a linear inequality, first graph the boundary line. Then decide which side of the line contains solutions to the inequality and whether the boundary line is included.

For an inequality with $y < \text{ or } y \leq$, shade below the line.

For an inequality with $y > \text{ or } y \geq$, shade above the line.

A dashed boundary line indicates that the line is not part of the solution.

A solid boundary line indicates that the line is part of the solution.

Choose a test point above or below the boundary line. The test point $(0, 0)$ makes the inequality true. Shade the region containing this point.

Check Skills You’ll Need

Solve each inequality. Graph the solution on a number line.

1. $12p \leq 15$
2. $4 + t > 17$
3. $5 - 2t \geq 11$
1–6. See back of book.

Solve and graph each absolute value equation or inequality.

4. $|4c| = 18$
5. $|5 - b| = 3$
6. $|2h| \leq 7$

Check Skills You’ll Need

Lessons 1-4 and 1-5

Objectives

1. To graph linear inequalities
2. To graph absolute value inequalities

Examples

1. Graphing a Linear Inequality
2. Real-World Connection
3. Graphing Absolute Value Inequalities
4. Writing Inequalities

Math Background

Graphing a two-variable inequality is the first step in learning how to graph a system of two-variable inequalities. Such systems play an important role in many applications, including maximizing and minimizing linear functions given a set of linear constants. This topic will be studied in Lesson 3-4.

More Math Background: p. 52D

Lesson Planning and Resources

See p. 52E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You’ll Need

For intervention, direct students to:

Solving Inequalities

Lesson 1-4: Example 1
Extra Skills and Word Problems Practice, Ch. 1

Absolute Value Equations and Inequalities

Lesson 1-5: Example 4
Extra Skills and Word Problems Practice, Ch. 1

Special Needs

Discuss the differences in the solutions of linear equations and linear inequalities. Illustrate how a linear inequality divides the coordinate plane into two half-planes, one of which is the solution region.

learning style: visual

Below Level

Tell students to look at the inequality symbol to determine whether the boundary line is dashed or solid. The line in the symbol should be a signal to draw a solid line.

learning style: visual
### 2. Teach

#### Guided Instruction

**Activity**
A vertical line in the coordinate plane will always intersect the graph of \( y = 2x + 3 \). Points on the vertical line that lie above the point of intersection have coordinates that satisfy \( y > 2x + 3 \). Points on the vertical line that lie below the point of intersection have coordinates that satisfy \( y < 2x + 3 \).

**1. Example**  
Alternative Method
You can also test points to decide which region to shade. Pick a point above the line and a point below the line. The coordinates of the point below the line will satisfy the inequality, but those of the point above the line will not. This tells you that the region below the line is the region to shade.

**2. Example**  
Teaching Tip
Quickcheck 2b provides a good opportunity to discuss how mathematical results must be interpreted in the light of restrictions imposed by a specific situation.

**Additional Examples**

1. Graph \( y > \frac{3}{2}x + 1 \).

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### 1 Example

**Graphing a Linear Inequality**

Graph the inequality \( y < \frac{1}{2}x - 3 \).

**Step 1** Graph the boundary line \( y = \frac{1}{2}x - 3 \). Since the inequality is less than, not less than or equal to, use a dashed boundary line.

**Step 2** Since the inequality is less than, \( y \)-values must be less than those on the boundary line. Shade the region below the boundary line.

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### 2 Example

**Real-World Connection**

**Entertainment** At least 35 performers of the Big Tent Circus are in the grand finale. Some pile into cars, while others balance on bicycles. Seven performers are in each car, and five performers are on each bicycle. Draw a graph showing all the combinations of cars and bicycles possible for the finale.

**Relate**
- the number of performers in cars
- plus
- the number of performers on bicycles

**Define**
- Let \( x \) = the number of cars.
- Let \( y \) = the number of bicycles.

**Write**
\[
7x + 5y \geq 35
\]

**Step 1** Find the intercepts of the boundary line. Use the intercepts to graph the boundary line.

When \( y = 0, 7x + 5(0) = 35 \).
\[
7x = 35 \quad x = 5
\]

When \( x = 0, 7(0) + 5y = 35 \).
\[
5y = 35 \quad y = 7
\]

Graph the intercepts (5, 0) and (0, 7). Since the inequality is greater than or equal to, use a solid boundary line.
**Lesson 2-7**

**Two-Variable Inequalities**

**Step 2** Choose a test point not on the boundary line. The test point (6, 4) makes the inequality true. Shade the region containing (6, 4).

The numbers of cars $x$ and bicycles $y$ are whole numbers. The situation is discrete. In the shaded region, all points with whole number coordinates (shown by dots) represent possible combinations of cars and bicycles for the grand finale.

### Quick Check

2. **Graphing Two-Variable Absolute Value Inequalities**

You can graph two-variable absolute value inequalities the same way you graph linear inequalities.

### Examples

**Example 1**

Graph each absolute value inequality.

- **a.** $y \leq |x - 4| + 5$
  
  Graph $y = |x - 4| + 5$.
  
  Since the inequality is **less than or equal to**, the boundary is solid and the shaded region is below the boundary.

- **b.** $-y + 3 > |x + 1|$
  
  Since the inequality is **less than**, the boundary is dashed and the shaded region is below the boundary.

### Quick Check

3. Graph each absolute value inequality.

- **a.** $y > -|x + 2| - 3$
  
  **See margin.**

- **b.** $2y + 3 \leq -|x - 5|$
  
  **See margin.**

**Lesson 2-7** Two-Variable Inequalities 103
You can write an inequality by examining a graph.

**Example 4** Writing Inequalities

**Multiple Choice** The graph is the solution of which inequality?

- A. \( y > |x - 3| + 2 \)
- B. \( y < |x - 3| + 2 \)
- C. \( y \geq |x - 3| + 2 \)
- D. \( y \leq |x - 3| + 2 \)

The boundary is \( y = |x - 3| + 2 \). The boundary is solid. The shaded region is above the boundary. This is the graph of \( y \geq |x - 3| + 2 \).

- The correct choice is C.

**Quick Check**

Write an inequality for each graph.

a. \( y \leq |x + 4| - 3 \)

b. \( y \geq 2x + 5 \)

**Exercises**

**Practice and Problem Solving**

For more exercises, see Extra Skill and Word Problem Practice.

**A. Practice by Example**

Example 1 (page 102)

- Graph each inequality. 1–9. See margin pp. 104–105.
- 1. \( y > 2x + 1 \)
- 2. \( y < 3 \)
- 3. \( x \leq 0 \)
- 4. \( y \leq x - 5 \)
- 5. \( 2x + 3y \geq 12 \)
- 6. \( 2y \geq 4x - 6 \)
- 7. \( y \geq \frac{2}{3}x + \frac{1}{3} \)
- 8. \( 3x - 2y \leq 9 \)
- 9. \( 5x \geq -y + 3 \)

Example 2 (pages 102–103)

10. **Cooking** The time needed to roast a chicken depends on its weight. Allow at least 20 min/lb for a chicken weighing as much as 6 lb. Allow at least 15 min/lb for a chicken weighing more than 6 lb. a–b. See back of book.

- a. Write two inequalities to represent the time needed to roast a chicken.
- b. Graph the inequalities.

Example 3 (page 103)

Graph each absolute value inequality. 11–19. See back of book.

- 11. \( y \geq |2x - 1| \)
- 12. \( y \leq |3x| + 1 \)
- 13. \( y \leq |4 - x| \)
- 14. \( y > |-x + 4| + 1 \)
- 15. \( y - 7 > |x + 2| \)
- 16. \( y + 2 \leq | \frac{1}{2}x | \)
- 17. \( 3 - y \geq |-x - 4| \)
- 18. \( 1 - y < |2x - 1| \)
- 19. \( y + 3 \leq |3x| - 1 \)
Write an inequality for each graph. The equation for the boundary line is given.

20. \( y = -x - 2 \)

21. \( 5x + 3y = 9 \)

22. \( 2y = |2x + 6| \)

Graph each inequality on a coordinate plane. 23–34. See back of book.

23. \( 5x - 2y \geq -10 \)

24. \( 2x - 5y < -10 \)

25. \( \frac{3}{4}x + \frac{2}{3}y > \frac{2}{3} \)

26. \( 3(x - 2) + 2y \leq 6 \)

27. \( 0.5x + 1.2y < 6 \)

28. \( -3x + 4y > -6 \)

29. \( \frac{1}{2}x + \frac{2}{3}y \geq 1 \)

30. \( |x - 1| > y + 7 \)

31. \( y - |2x| \leq 21 \)

32. \( \frac{2}{3}x + 2 \leq \frac{7}{2}y \)

33. \( 0.25y - 1.5x \leq -4 \)

34. \( 6x - 4y \geq -3 \)

35. Open-Ended Write an inequality that has \((10, 15), (-10, 20), (-20, -25),\) and \((25, -10)\) as solutions. Answers may vary. Sample: \( y \leq -\frac{5}{3}x + \frac{95}{3} \)

Write an inequality for each graph.

36. \( x > -3 \)

37. \( y \leq \frac{2}{3}x + 2 \)

38. \( y \geq -2x + 4 \)

39. \( y \leq |x + 2| \)

40. \( y < -|x - 4| \)

41. \( y > |x + 1| - 1 \)

42. Multiple Choice Which graph best represents the solution of the inequality \( y \geq 2|x - 1| - 2? \) C

43. Writing When you graph an inequality, you can often use the point \((0, 0)\) to test which side of the boundary line to shade. Describe a situation in which you could not use \((0, 0)\) as a test point. Answers may vary. Sample: when it lies on the boundary line.

Graph each inequality on a graphing calculator. Then sketch the graph.

44. \( y \leq |x + 1| - |x - 1| \)

45. \( y > |x| + |x + 3| \)

46. \( y < |x - 3| - |x + 3| \)

47. \( y < 7 - |x - 4| + |x| \)

44–47. See back of book.
Graph each inequality.
1. \( y \geq -\frac{1}{2}x + 3 \)

2. \(-y + 1 > -|x + 3|\)

**Alternative Assessment**
Have students work in pairs. Each student gives the coordinates of two points not on the same vertical line. The partner writes and graphs a linear inequality whose boundary contains the given points. Each partner checks the other's work.

**Test Prep**

**Multiple Choice**

48. The graph at the right shows which inequality?
   A. \( y > |x + 4| - 4 \)
   B. \( y > |x - 4| + 4 \)
   C. \( y < |x + 4| - 4 \)
   D. \( y < |x - 4| + 4 \)

49. The graph of which inequality has its vertex at \((2\frac{1}{2}, -5)\)?
   J. \( y < |2x - 5| + 5 \)
   G. \( y < |2x + 5| - 5 \)
   H. \( y > |2x + 5| - 5 \)
   J. \( y > |2x - 5| - 5 \)

50. Which inequality is NOT equivalent to the others?
   D. \( y = 2x - 9 \)
   B. \( 3y = 2x - 9 \)
   C. \( 2x - 3y = 9 \)
   D. \( 2x - 3y = 9 \)

51. The graph at the right shows which inequality?
   F. \( y = -2.5x + 5 \)
   G. \( 2.5x + y > 5 \)
   H. \( -2.5x + y < 5 \)
   J. \( 5x + 2y \leq 5 \)

52. Which point(s) are solutions of the inequality \(5x + 3y \geq 2\)?
   C. \((0, 0)\) and \((-1, 1)\) and \((2, -1)\)
   A. I only
   B. I and II
   C. III only
   D. II and III

**Short Response**

53. At least 300 tornadoes occur in the United States each year. Write an inequality to model the number of tornadoes that could occur during the next \(x\) years. Describe the domain and range of the inequality.
   See margin.

**Mixed Review**

**Lesson 2-6**
Graph each function by translating its parent function.
54. \( y = 2x + 5 \)
55. \( y = |x| - 3 \)
56. \( f(x) = |x + 6| \)

57. \( f(x) = x - 2 \)
58. \( y = |x + 2| \)
59. \( y = |x - 1| + 5 \)


**Lesson 2-3**
Determine whether \(y\) varies directly with \(x\). If so, find the constant of variation.
   yes; yes; yes;
60. \( y = x + 1 \) no
61. \( y = 100x \) 100
62. \( 5x + y = 0 \) yes
63. \( y - 2 = 2x \) no
64. \( x = \frac{y}{3} \) yes; 3
65. \( -4 = y - x \) no
66. \( y = -10x \) yes; 67. \( xy = 1 \) no
68. \( xy = 100 \) no

**Lesson 2-2**
Graph each pair of equations on the same coordinate plane.
69. \( y = x, y = -x + 5 \)
70. \( y = -2x + 1, y = 2x \)
71. \( y = 4x - 1, y = x \)

**Exercises**

53. [2] \( y \geq 300x \), where \( y \) is the number of tornadoes that could occur in the next \(x\) years. The domain is all whole numbers greater than 0, since years are whole numbers and can't be negative. The range is whole numbers \( \geq 300 \), since you cannot have a fraction of a tornado and in the first year you will have at least 300.

[1] includes only inequality with no explanation of domain and range